Integrating market correlation into risk-adjusted return

In the March issue of *Risk Management for Investors*, Marc Goodman, Kenneth Shewer and Richard Horwitz presented findings from their research on hedge fund diversification. In this article, they propose an 'enhanced Sharpe ratio' that supports their rationale

Institutional and high-net-worth investors are increasingly considering alternative investments, as their expectations for returns from equities range from low to bleak. They are looking for investments that are not correlated with the equity market, that is, investments that can perform well in any environment, but particularly when their traditional long-only equity investments are not producing adequate returns. This is difficult because, as the world has become more global, international markets have become more correlated. Diversification is thus more difficult to achieve, and it is more valuable than ever.

If you are an investor with a significant exposure to the S&P 500 through traditional investments, which of the following hedge funds would be the most attractive to you as an incremental investment (assume you keep your original S&P 500 exposure, and that the S&P 500 earns a 12% return, the risk free rate is 5% and the standard deviation of each of the hedge funds and the S&P 500 is 18%):

- Hedge fund A: Has an expected annualised return of +12% and is perfectly correlated with the S&P 500.
- Hedge fund B: Has an expected annualised return of +8% and is uncorrelated (100% independent) with the S&P 500.
- Hedge fund C: Has an expected annualised return of –2% and is perfectly negatively correlated with the S&P 500.

The correct answer is all of the above. An investor with significant exposure to the equity market should have no preference when faced with these alternative investments[1].

Our research on hedge fund diversification[2] made two key conclusions with respect to the correlation of hedge funds to the equity market:

- While hedge funds can provide valuable diversification from the equity market, the performance of most hedge funds is highly correlated with the equity market.
- Traditional measures of risk and return (eg, the Sharpe ratio) do not differentiate between risk that is correlated with the equity market – to which most investors have significant exposure via traditional investments – and risks that are not correlated with the equity market.

This research sent us in search of a measure of risk-adjusted return that, in addition to adjusting returns for volatility, as the Sharpe ratio does, also adjusts returns for correlation to the equity market (beta). This has lead to the development of the Bavar (beta and volatility adjusted returns) ratio. The Bavar ratio adjusts the beta of various investments to be equivalent, so that a fund that has a lower return but is uncorrelated to the market can be appropriately compared with a fund that achieves a higher return but is highly correlated to the market. This article will assume the 'market' to be the S&P 500. However, the methodology will work equally well for any other market.

For the statistical derivation of our Bavar ratio, we base our formulation on the Sharpe ratio:

$$\frac{\{return\ of\ fund\} - \{risk\text{-}free\ rate\}}{\sigma\{return\ of\ fund\}}$$

Because volatility and correlation are non-additive, we cannot compare two funds with different correlations to the S&P 500 (beta) simply by subtracting the exposure of each to the S&P 500. However, we do know how return, volatility and correlation combine. Using this knowledge, our methodology brings parity, and therefore comparability, to hedge funds with different levels of correlation with the S&P 500 by normalising the beta of each investment to one. We do this by combining each hedge fund with the appropriate level of long/short exposure to the S&P 500, in order to bring the hedge fund's total S&P 500 exposure to a beta of one. We then recalculate the new return and new volatility of the beta-adjusted hedge fund. The results are then plugged into the Sharpe ratio to create the Bavar ratio:

$$\frac{(\{return\ of\ fund\} - \{risk\text{-}free\ rate\}) + (1-\beta) \times (\{return\ of\ S\&P\} - \{risk\text{-}free\ rate\})}{\sqrt{(1-R^2) \times \sigma^2\{return\ of\ fund\} + \sigma^2\{return\ of\ S\&P\}}}$$

where σ^2 {*return of fund*} is the variance of the return of the funds; σ^2 {*return of S&P*} is the variance of the return of the S&P; R^2 is the coefficient of determina-

[1] *It is not intuitive that one should have no preference between:*

- *Hedge fund A: an investment that is perfectly positively correlated with the S&P 500 (beta of one) and earns an expected return of +12% with a volatility of 18%; and*
- *Hedge fund C: an investment that is perfectly negatively correlated to the S&P 500 (beta of minus one) but earns an expected return of –2% with a volatility of 18%.*

The reason is that combining a long-only equity portfolio with hedge fund A results in an identical risk-adjusted return to the combination of the long-only equity portfolio with hedge fund C. The former would, of course, have a higher expected return and a higher expected risk than the latter, but on a risk-adjusted basis, the expected returns would be identical.

It is worth noting that one could also equalise the return and risk of the combined portfolios by equalising the return and risk of hedge fund C to hedge fund A. This is accomplished by increasing the S&P 500 exposure (representing a beta of two) in hedge fund C. The added S&P 500 exposure would earn 14% (twice the equity risk premium of 7%). When added to the original return of hedge fund C of –2%, it now returns 12%, which is equal to that of hedge fund A.

[2] *Squeezing the Best from Hedge Fund Diversification, Risk March 2002, pages 82–85*

tion of the regression of the {*return of fund*} with the {*return of S&P*}; and β is the coefficient of the {*return of the S&P*} in that regression.

The amount of additional S&P 500 exposure required to bring the hedge fund to a beta of one would be $(1 - \beta)$, where β is the coefficient of the regression of the returns of the fund as the dependent variable and that of the S&P 500 as the independent variable. The capital asset pricing model tells us that the expected return generated from this amount of additional S&P 500 exposure would be $(1 - \beta)$ multiplied by the equity risk premium, the difference between the return of the S&P 500 and the risk-free rate. Consequently, the return of the beta-adjusted hypothetical hedge fund would be the combined return of the hedge fund and this additional exposure to the S&P 500, as shown in the numerator above.

The returns of a hedge fund combined with $(1 - \beta)$ exposure to the S&P 500 is equal to the residuals of the regression that calculated the beta above combined with the S&P 500. We use this fact to facilitate the calculation of the volatility of the beta-adjusted hedge fund. However, the residuals of the regression are independent of the S&P 500, and therefore the variance of the beta-adjusted hedge fund is equal to the sum of the variance of the residuals and that of the S&P 500. By definition, the coefficient of determination (R^2) of the regression is $1 - \sigma^2$\{*residuals*\}/σ^2\{*return of fund*\}. Consequently, the variance of the residuals, σ^2 \{*residuals*\}, is equal to $(1 - R^2) \times \sigma^2$ \{*return of fund*\}. The standard deviation of the beta-adjusted hedge fund is the square root of the sum of the variance of residuals and the variance of the S&P 500. The denominator of the Bavar ratio is determined by plugging the standard deviation of the beta-adjusted hedge fun into the Sharpe ratio formulation, as shown above.

Key conditions

We vetted the Bavar ratio by testing that it worked for the following key conditions:

□ Perfect positive correlation to the S&P 500 (hedge fund A, above). In this case, $R^2 = \beta = 1$ and the Bavar ratio simplifies to the Sharpe ratio. Hedge fund A thus has a Bavar ratio of:

$$\frac{(12\% - 5\%)}{18\%} = \frac{7\%}{18\%} = 0.39$$

□ Perfect negative correlation to the S&P 500 (hedge fund C, above). In this case, $R^2 = 1$ and $\beta = -1$, and the Bavar ratio again simplifies to the Sharpe ratio. Hedge fund C thus has a Bavar ratio of:

$$\frac{(-2\% - 5\%) + \left[(1+1) \times (12\% - 5\%)\right]}{18\%}$$
$$= \frac{7\%}{18\%} = 0.39$$

□ Uncorrelated (statistically independent) to the S&P 500 (hedge fund B, above). In this case, $R^2 = \beta = 0$. In addition to the return generated by the hedge fund, the beta-adjusted hedge fund earns the equity risk premium from the additional exposure to the S&P 500. Because the returns of the hedge fund and the returns of the S&P 500 are independent, the standard deviation of the beta-adjusted hedge fund is equal to the sum of the variances of the two. Assuming that the volatility of the hedge fund equals that of the S&P 500, the standard deviation of the beta-adjusted fund will equal the square root of two times the volatility of the S&P 500. Hedge fund B thus has a Bavar ratio of:

$$\frac{(8\% - 5\%) + (1 - 0) \times (12\% - 5\%)}{\sqrt{(1-0) \times 18\%^2 + 18\%^2}}$$
$$= \frac{10\%}{25\%} = 0.39$$

When using monthly returns to calculate the Bavar ratio, it will be subject to the same, and no greater, data problems as the Sharpe ratio, such as limited data, the length of actual records varying across funds, changes in the underlying portfolios, and the hedge fund manager's ability to 'manage' monthly valuations. The Bavar value may be maximised when the underlying investments in hedge funds are processed through a risk management system that maps positions to risk factors. The factor and idiosyncratic risk can then be appropriately isolated from the beta risk (see our earlier article), permitting a more rigorous quantification of the Bavar ratio.

Use of the Bavar ratio can significantly enhance an investor's ability to construct risk-efficient portfolios by providing a methodology of appropriately comparing the risk-adjusted returns of funds that have varying correlations to the S&P 500 (varying betas). Investing in hedge funds that are not market neutral and have a positive beta can make sense, as long as the higher correlation to the equity market is appropriately compensated by higher returns. If the risk-reward is justified, the investor can simply hedge out the market exposure, resulting in an investment with attractive alpha, known as 'alpha transport'. Bavar provides a holistic framework to support this decision. □

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